# Lie Groups and Lie Algebras-MMath-2021
# Final test

**Instructions:** Total time 3 hours. Extra 30 minutes to upload the solutions on Moodle. Maximum score is 40. You are allowed to use only terms, definitions and results covered in the class. Results covered in the class may be quoted and used without proof, if you use exercises, you must also include solutions. You may reach me at maneesh.thakur@gmail.com, Phone: 9650330176

1. Give an example (with justification) of a representation of the Lie group $(\mathbb{R}, +)$ which is not completely reducible. (10)
2. Prove that the standard representation $SO(3; \mathbb{R}) \to GL(3; \mathbb{R})$ given by the inclusion, is irreducible. Is the complex representation $SO(3; \mathbb{R}) \to SO(3; \mathbb{C})$ given by the inclusion map irreducible? Explain. (10)
3. Let $G$ be a connected Lie group and $\pi : G \to GL(V)$ a representation. Prove that for a nonzero vector $v \in V$, $\pi(g)v = v$ for all $g \in G$ if and only if $d\pi(e)(X)v = 0$ for all $X \in \text{Lie}(G)$. (10)
4. Let $L$ be a nonzero nilpotent real Lie algebra. Prove that $I \cap Z \neq 0$ for any nonzero ideal $I$ of $L$, where $Z$ is the center of $L$. Prove that if $G$ is a nontrivial connected nilpotent Lie group, its center $Z(G)$ has positive dimension. (10)